Pages 1–3 involve small errors, then come some big ones

Page vii, line 4− [4 lines from the bottom]
"Shifin" should be "Shifrin"

Page 13, line 5−
"length t" should be "length Δt"

Page 17, line 4−
"mass of of" should be "mass of"

Page 24, line 11−
"understood" should be "understand"

Page 29, line 10
"every" should be "ever"

Page 32, line 10
"from it's consequence" should be "from its consequence"

Page 35, line 6−
there is extra space before the comma at the end of the line

Page 44, Problem 12.
See page 3 of this errata for a reference to the history of this problem.

Page 55, second line after the figure
"planet it always" should be "planet is always"

Page 67, line 3− (not counting footnote)
"know" should be "known"

Page 82, line 9
this line should be replaced by

in a circle of radius $|\mathbf{p}| \cdot \sin\theta$, where θ is the angle between $\mathbf{w}$ and $\mathbf{p}$, and $X_{\mathbf{p}}$ has length $|\mathbf{p}| \cdot |\mathbf{w}| \cdot \sin\theta$. Thus,

Page 96, equation (1)
the numerator on the right should be $m_1v_1 + m_2v_2$

Page 112, line 13−
$\frac{1}{6}$ should be $\frac{1}{6}m$

Page 115, first figure

$\mathbf{w}_1{}^*$ and $\mathbf{v}_1{}^*$ should be $\mathbf{w}_1^*$ and $\mathbf{v}_1^*$

Page 119, line 4

"form" should be "from"

Page 137, last line of text

"AC and D" should be "AC and BD"

Page 189, line 4

"of its eigenvalues" should be "of its eigenvectors"

Page 200, line 14

"that is an only" should be "that is only"

Page 214, line 8–

"principal" should be "principle"

Page 215, line 2–

"stationery" should be "stationary"

Page 318, line 6–

"is a simply" should be "is simply"

Page 326, line 4

$\sin vvt$ should be $\sin vt$

Page 373, Problem 5.

This problem should have 

before it.

Page 462, line 12, formula for $J(f)$ should be

$$J(f) = \int_a^b F(f(t), f'(t), t)\,dt$$

Page 466, lines 7, 17, 18

"principal" should be "principle"

Page 475, first displayed equation should be

$$\phi_s(x^a) = A(s)(x^a)$$

Page 478, line 6–SW

there should be no period after the word "commensurable"

Page 522, line 10

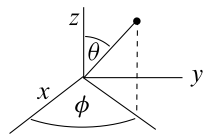
"end of from these" should be "end of these"

Page 536, line 7

there is an extra space between “576” and the right parenthesis

Page 539, Problem 1(d)

The equation should read $y = r \sin\theta \sin\phi$ and the figure should be



Page 568, line 7

^{t} should be $^{\mathbf{t}}$

Page 617, line 5, and displayed equation (5) following it

“is can be” should be “can be” and in the displayed equation the lower limit on the integral sign should be $(\mathring{q}, \mathring{p}, t_0)$

Page 595, second equation in 1.(a)

should be $L_X(f\lambda) = Xf \cdot \lambda + f \cdot L_X\lambda$

Page 626, lines 7 and 8

$\rho\colon \mathbb{R} \to S^1$ should be $\rho\colon \mathbb{R}^2 \to S^1 \times \mathbb{R}$

$\tilde{\gamma}\colon [a, b] \to \mathbb{R}$ should be $\tilde{\gamma}\colon [a, b] \to \mathbb{R}^2$

Page 656, line 4− (after footnote)

PA should be BA

Page 701

The entry Newton, I. [1], *The Correspondence of Isaac Newton*, should include “H. W. Turnbull, ed.”

Page 703

Before the last line, insert the line

Steele, William John *See* Tait, Peter Guthrie.

Page 704

add the reference

Tait, Peter Guthrie and Steele, William John
[1] *A Treatise on Dynamics of a Particle*, 7th ed., Macmillan and Co. 1900.

I am grateful to Mark Villarino Bertram for apprising me of this book, which seems to be the ultimate source of many of the problems to be found in books on mechanics. It can be read online at

`catalog.hathitrust.org/Record/005858641`.

Big Errors

Page 586, lines 4 to 6, last sentence should read

For the moment, we simply note that a type 2 generating function gives a canonical transformation $(q, p) \mapsto (Q, P)$ for which the Q^i are integrals.

Page 586, lines 10$-$ to 8 $-$, should read

Analogous to the situation for a type 2 generating function, if there is a type 1 generating function, then there is a canonical transformation $(q, p) \mapsto (Q, P)$ for which the P_j are integrals.

Page 603, the part of the statement of Corollary 2 following the comma should read

for which Q^α and P_β are integrals for $\alpha \in \mathcal{I}_Q$ and $\beta \in \mathcal{I}_P$.

Page 614, the first paragraph of **Functions in involution** should read

Functions in involution. In all the solvable cases of mechanics on $T^*M^n \times \mathbb{R}$ we can find n integrals $f_1, \dots, f_n$ which are also **in involution**, that is, their Poisson brackets $\{f_i, f_j\} = 0$. Examples are given on pages 627 and 628, and we note more generally that solving a problem using type 1 generating functions gives integrals Q^i which are part of a canonical coordinate system (Q, P), so these Q^i are also in involution (page 607). A similar result holds when we use a type 2 generating function, or even a generating function that is a mixture of type 1 and type 2 generating function, as in Corollary 2 on page 603.

* *

Finally, several critical queries by Marco Aurelio García Portillo led me to realize that everything on pages 632, 633, and 634, up to "At the end of all this ... are solvable by quadratures" is essentially gibberish, and should be replaced by the following two pages (leaving, for now, the top part of page 634 an embarrassing blank). And, on page 637, in the statement of the Action-Angle Variables Theorem, the phrase "(automatically ... as on page 634)." should simply be eliminated

Thus, we have replaced the invariants f_i with the more geometrically defined action variables $J_1, \ldots, J_n$, and we would hope that we can now extend them to a geometrically defined canonical coordinate system

$$(\varphi, J) = (\varphi^1, \ldots, \varphi^n, J_1, \ldots, J_n),$$

with the φ^i analogous to those in the invariant tori theorem.

Of course, we are no longer working on an arbitrary symplectic manifold, but specifically on the cotangent bundle T^*M, since we are using the 1-form

$$\boldsymbol{\theta} = \sum_{k=1}^{n} p_k \, dq^k.$$

This restriction certainly won't be bothersome for the mechanics examples that we will first be looking at, so, saving the general treatment for later, we outline a proof for the case of T^*M.

In addition, we are going to assume that one of the f_i, say f_1, is just H. This is not that restrictive, since the invariance of the f_i automatically implies that $\{f_i, H\} = 0$, and in mechanics problems H is always the first integral we consider. Including H in our set of f_i insures that H is constant on each of the tori we construct (which is generally true even without making this choice, see the remark starting at the bottom of page 636).

The proof will be rather analogous to that of the classical Liouville theorem, although we will sort of be working backwards. We are also going to need one additional hypothesis, that (q, J) is a coordinate system [this corresponds to $(*)$ in Liouville's theorem; it wasn't required as an assumption when we were working locally, but now we are working "semi-globally", in a neighborhood of one of the invariant tori]. The n-tuples $J = (J_1, \ldots, J_n)$ will now serve as coordinates for the various tori $C_{\mathbf{b}}$, and we will let $C_{\mathring{J}}$ denote the invariant torus determined by a particular value $\mathring{J}$ of J.

Since we are assuming that (q, J) is a coordinate system, the implicit function theorem gives us a function $\widehat{p}$ such that,

$$J\big(\mathring{q}, \widehat{p}(\mathring{q}, \mathring{J})\big) = \mathring{J} \qquad \text{for all values } \mathring{q} \text{ and } \mathring{J} \text{ of } q \text{ and } J.$$

Also, as in the proof of the product neighborhood lemma, we choose initial points $\wp_{\mathring{J}}$ in $C_{\mathring{J}}$ lying along an n-dimensional submanifold transversal to all the tori C_J.

We now define a function S by

$$S(\mathring{q}, \mathring{J}) = \int_\gamma \sum_{k=1}^{n} p_k \, dq^k = \int_\gamma \boldsymbol{\theta} \qquad \begin{array}{l}\text{where } \gamma \text{ is a curve lying entirely in } C_{\mathring{J}},\\ \text{going from } \wp_{\mathring{J}} \text{ to } \big(\mathring{q}, \widehat{p}(\mathring{q}, \mathring{J})\big).\end{array}$$

Obviously $S(\mathring{q}, \mathring{J})$ is multiple-valued, since γ can wrap around the torus any number of times. But we still have

(a) $$\frac{\partial S}{\partial q^i}(q, J(q,p)) = p_i.$$

Notice that, except for the multiple-valuedness, this is precisely equation (6) on page 617, the first step in using S to obtain a type 2 generating function, with the required P_i being the J_i. The corresponding Q^i, which we didn't bother to identify in the proof of Liouville's theorem, are given by equations (F_2-b) on page 580; calling them φ^i instead of Q^i, the equations become

(b) $$\frac{\partial S}{\partial J_i}(q, J(q,p)) = \varphi^i.$$

The φ^i are now multiple-valued, but using the fact that each $C_{\mathbf{b}}$ is isotropic, we see that dS is a well-defined 1-form on each torus, and then on each torus we have

$$\int_{\gamma_j} d\varphi^i = \int_{\gamma_j} d\left(\frac{\partial S}{\partial J_i}\right) = \frac{\partial}{\partial J_i}\int_{\gamma_j} dS = \frac{\partial}{\partial J_i}(2\pi J_j) = 2\pi\delta_i^j,$$

so that the φ^i are in fact well-defined multiple-valued functions mod 2π.

Equations (a) and (b) thus give us a canonical transformation $(q,p) \mapsto (\varphi, J)$, so the (φ, J) are also canonical coordinates, known as **action-angle** variables. [It shouldn't be surprising that the J_i are thus in involution, since the f_i are, and the J_i depend only on the f_i, not on the φ^j.]

Now (compare pages 587–588 and 619), since H is constant on each torus, and thus a function only of the $J_1, \dots, J_n$, independent of the φ^i, Hamilton's equations

$$\dot{\varphi}^i = \frac{\partial K}{\partial J_i}, \qquad \dot{J}_i = -\frac{\partial K}{\partial \varphi^i},$$

for K, the Hamiltonian H in (φ, J) coordinates, will reduce to

$$\dot{\varphi}^i = \nu_i(J), \qquad \dot{J}_i = 0,$$

with solutions (in condensed notation)

$$J_i = a_i, \quad \varphi^i(t) = b_i + \nu_i(a_1, \dots, a_n)\cdot t \qquad \text{for constants } a_i, b_j.$$

At the end of all this, note that S is defined as an integral, and hence the φ^i and our equations are solvable by quadratures. **Q.E.D.**