

Page viii, paragraph 2, line 6– [6 lines from the bottom]:

“Needles” should be “Needless”

Page 28, line 2:

“natural number.” should be “natural number (or 0).”

Page 29, first line of Problem 7:

$\sum_{i=1}^n$ should be $\sum_{k=1}^n$

Page 33, Problem 22:

Eliminate everything in (a), change (b) to (a) and (c) to (b), and add the following remarks:

This elegantly tricky proof, due to Cauchy, avoids difficulties arising in more straightforward approaches, see the Wikipedia article “Inequality of arithmetic and geometric means”, where this proof is described as using “forward-backward induction”.

Another proof is given in Problem 35 of Chapter 18.

Page 35, change last sentence of the first paragraph to:

Tradition called for the error to be mentioned to all other members of the department, except that the actual author of the error was kept ignorant of the fact, in order to forestall any resignations.

Page 35, at end of Problem 27 add:

(The answer to this Problem, on page 622, contains some remarks on its history.)

[The answer should appear in the Answer section, rather than the separate Answer Book, since Problem 28 suggests that one might want to look it up.]

Page 46, line 9:

$$f(x) = x^2 + 3x + 3 - 3(x + 1)$$

Page 48, Problem 4, line 2:

“you” should be “your”

Page 65, line 12:

“Out last” should be “Our last”

Page 72, line 6 of Problem 16:

“cases where A and B ” should be “cases where A and C ”

Page 75, starting at line 4:

For convenience, we will use analogous notation for any point of the plane described by a single letter. Thus, if we ...

Page 75, line 10:

“we’ve” should be “we”

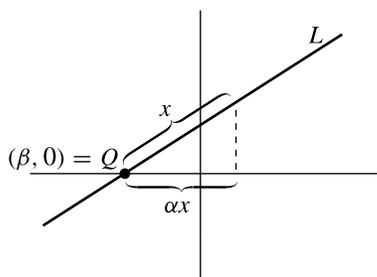
Page 79, Problem 4, line 7:

The (a) should be eliminated, since there is no (b).

Page 81, at end of first paragraph add:

(A geometric treatment, perhaps easier to follow, appears in the problems for this Appendix.)

Page 81, Figure 5 should be:



Page 81, lines 13– to line 11– should read:

can be written in the form

$$\alpha x + \beta$$

for an appropriate α . On the other hand, ...

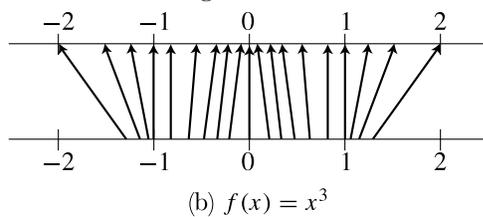
Page 84, Figure 2:

The θ_2 in the legend should be θ_0

Page 88, Problem 6:

In the second line, $y = a$ should be $x = a$, and the same change should be made in Figure 7.

Page 91, needs a better version of Figure 2b:



(b) $f(x) = x^3$

Page 94, paragraph beginning “The trick in this case”:

For more clarity, in the third line “ x be less than half as far from 0 as a ” should be “ x be less than half as far from a as a is from 0”

Page 109, first line of Problem 15 should end:

... of the number $\alpha = \lim_{x \rightarrow 0} (\sin x)/x$ (this limit is > 0).

Page 112, the first line of Problem 33 should be:

For each of the following, find the limit, or show that it does not exist.

Page 116, line 4:

“(if fact” should be “(in fact”

Page 119, line 3:

“There theorems” should be “These theorems”

Page 139, line 6:

“Chapter 6” should be “Chapter 5”

Page 140, line 5:

$f(x) <$ should be $f(x) \leq$

Page 141, Problem 11(c):

Mark Villarino Bertram kindly pointed out to me that the remark “By calculating the areas of polygons ... Archimedes [showed] that $\frac{223}{71} < \pi < \frac{22}{7}$ ”, an idea which I probably parroted from some book, is completely incorrect. Archimedes never even mentions areas; all of his computations are made for perimeters, with very delicate rounding techniques used for the final computation. He also recommends the paper *Archimedes and the Measurement of the Circle: a New Interpretation*, by W. Knorr, in *Archive for the History of the Exact Sciences*, Vol. 15, No. 2, pp. 115–140, which argues that the extant version of Archimedes’ work is a corrupt extraction from a comprehensive investigation concerning convergence rates of different approximation techniques.

Page 174, line 11:

“presumable” should be “presumably”.

Page 194, line 14:

$$h(b) = f(b) - \left[\frac{f(b) - f(a)}{b - a} \right] (b - a)$$

Page 198, line 3—:

“be” should be “by”.

Page 207, last line of Problem 8:

$(Ax_0 + By_0 + C)$ should be $|Ax_0 + By_0 + C|$

Page 213, Problem 52 should have:

(ii) $\lim_{x \rightarrow 0} \frac{\cos^2 x - 1}{x^2}$, in terms of $\lim_{x \rightarrow 0} (\sin x)/x$.

Page 215, Problem 65:

$(1 + x)^n > 1 + nx$ should be $(1 + x)^n \geq 1 + nx$.

Page 217, Problem 69, next to last line of (a):

$0 < b$ should be $0 \leq b$

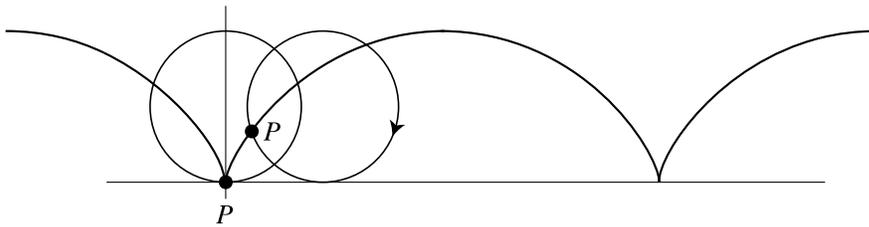
Page 227, Problem 2:

“Figure 30” should be “Figure 31”

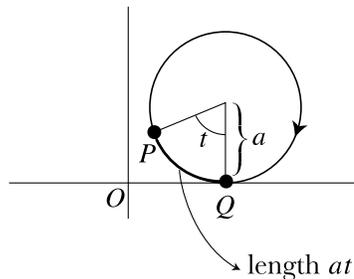
Page 248, 2 lines before the problem section:

“Problems 2 and 4” should be “Problems 2 and 6”

Page 250, Figure 9 should be replaced by



Page 251, Figure 10 should be replaced by



Page 251, last display should be:

$$u(t) = 2\pi na \pm \left\{ a \arccos \frac{a - v(t)}{a} - \sqrt{[2a - v(t)]v(t)} \right\}.$$

Page 277, Problem 23(d):

“for some ξ in $[a, b]$.” should be “for some ξ in $[a, b]$ (assume that fg is integrable on $[a, b]$, as shown in Problem 38).”

Page 281, Problem 39(c):

“If equality holds,” should be “If equality holds and $\int_a^b g^2 > 0$,”

Page 288, line 10—:

$n \neq -1$ should be $n \neq 1$

Page 298, Problem 8 should begin:

Find $F'(x)$ if $F(x) = \int_0^x xf(t) dt$, for a continuous f .

Page 301, line 3:

“ $f(x) = (ax + b)/(cx + d)$.” should be $f(x) = (ax + b)/(cx + d)$, provided that $ad \neq bc$.”

Page 316, Problem 4(d), last line:

“up” should be “up (or down)”

Page 316, Problem 5, first line:

“ $f(x) = a/\theta$ ” should be “ $f(\theta) = a/\theta$ ($a > 0$)”

Page 320, Problem 24:

“ $0 \leq x \leq \pi$.” should be “ $0 \leq x \leq \pi$ and $x \neq \pi/2$.”

Page 327, line 13 should read:

that precisely one property of π is essential (together with $\sin^2 + \cos^2 = 1$ and its consequences)—

Page 328, Problem 1(b) should begin:

(b) Let P_m be the regular polygon of m sides inscribed in the unit circle, with area A_m . For $m > 3$, show that

Page 342, Proof of Corollary 2:

“Let” should be “Left”

Page 362, Problem 48 should begin:

Suppose that g_1, g_2, g_3, \dots are continuous functions with $\lim_{x \rightarrow \infty} g_i(x) = \infty$.

Page 364, line 3—:

‘by’ should be ‘be’

Page 380, first equation should be:

$$\int \frac{1}{(x^2 + x + 1)^2} dx = \frac{16}{9} \int \frac{1}{\left[\left(\frac{x + \frac{1}{2}}{\sqrt{\frac{3}{4}}} \right)^2 + 1 \right]^2} dx.$$

Page 387, Problem 12(2):

change $-$ to $+$.

Page 392, line -2 :

formula should be replaced by $s_n b_{n+1} + \sum_{k=1}^n s_k (b_k - b_{k+1}) = \sum_{k=1}^{n-1} s_k (b_k - b_{k+1}) + s_n b_n$

Page 394, line 5—:

“Harold” should be “Harald”

Page 395, line 3—:

$$\frac{2^n}{2n+1} \frac{(n!)^2}{(2n)!} \text{ should be } \frac{2^{2n}}{2n+1} \frac{(n!)^2}{(2n)!}$$

Page 398, line 5—:

$f(ax) - f(\beta x)$ should be $f(\alpha x) - f(\beta x)$

Page 404, Figure 9(b):

The label s_2 should just be s

Page 404, line 8—:

we conclude that if f' is continuous, then the surface area is

Page 408, line 2:

“radius” should be “diameter”

Page 408, Problem 12:

“Problem 19-3” should be “Problem 3 on page 405” and similarly for “19-4”

Page 414, line 3—:

$$P_{2,0}(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 = 1 + x + \frac{x^2}{2}$$

Page 421, line 5:

$$\arctan^{(0)}(0) + \arctan^{(1)}(0)x + \frac{\arctan^{(2)}(0)}{2!}x^2 + \cdots + \frac{\arctan^{(2n+1)}(0)}{(2n+1)!}x^{2n+1},$$

Page 423, line 6:

$$m \int_a^x (x-t)^n dt \leq R_{n,a}(x) \leq M \int_a^x (x-t)^n dt,$$

Page 423, line 11:

$$R_{n,a}(x) = \frac{f^{(n+1)}(t)}{n!} \frac{(x-a)^{n+1}}{n+1} = \frac{f^{(n+1)}(t)}{(n+1)!} (x-a)^{n+1},$$

Page 434, Problem 9(c):

Delete the last three lines.

Page 436, Problem 18, second line should begin:

the remainder—the integral form or Lagrange form on page 423, or the Cauchy form at the end of Problem 19(b).

Page 443, line after equation (*):

In order to reach a contradiction we will then find

Page 448, Problem 3(c): The $>$ symbols in the last two lines should be \geq .

Page 503, Proof of Theorem 1:

In the multi-line formula after “Thus, if $n > N$ we have”, the second line should read

$$\leq \int_a^b |f(x) - f_n(x)| dx \quad (\text{Problem 13-37})$$

Page 599, Problem 2:

It was pointed out to me by Richard Ferraro that the method outlined for this problem is unduly optimistic. The definition of \leftarrow needs to be modified to allow for negative numbers, and a proof of the associative law for \oplus looks treacherously difficult. However, a complete elegant construction is give in the book

A. H. Lightstone, *Symbolic logic and the real number system*.

Page 611, line 5—:

Many of the examples . . .

Page 612, after reference [23]:

Kevin Gallagher pointed out to me that *Problems in Calculus and Analysis* by Albert A. Blank is a companion to Volume 1 of [23], including additional exercises and problems, as well as answers and solutions to many of the problems in Volume 1. (Unfortunately, Blank’s book is out of print and very hard to find.)

Page 615, line 8:

As might be inferred from the quotation on page 38, . . .

Page 622, add the following answer for Problem 27 of Chapter 2.

27. Everyone resigned on the seventeenth luncheon meeting.

The reasoning is as follows (for the sake of sanity, “he or she” shall be rendered as “he” throughout). First suppose there were only 2 professors, Prof. A and Prof. B, each knowing of the error in the other’s work, but unaware of any error in his own. Then neither is surprised by Prof. X’s statement, but each *expects the other to be surprised*, and to resign at the first luncheon meeting next year. When this doesn’t happen, each realizes that this can only be because he has also made an error. So at the next meeting, both resign.

Next consider the case of 3 professors, Profs. A, B and C. Prof. C knows that Prof. A is aware of an error in Prof. B’s work (either because Prof. A found the error and informed him, or because he found the error and informed Prof. A). Similarly, he knows that Prof. B knows that there is an error in Prof. A’s work. But Prof. C thinks he has made no errors, so as far as he is concerned, the situation vis-a-vis Profs. A and B is precisely that analyzed in the previous paragraph (Prof. C is assuming, of course, that no one believes an error to exist when one doesn’t). So Prof. C expects both Prof. A and Prof. B to resign at the second meeting. Of course, Profs. A and B similarly expect the other two to resign at the second meeting. When no one resigns, everyone realizes that he has made an error, so all resign at the third meeting.

Now you can turn this into a proof by induction (can’t you?).

Littlewood’s *A Mathematician’s Miscellany* ([12] in the Suggested Reading, available on-line) relates what he claims is the original form of the puzzle.

Three ladies, A, B, C in a railway carriage all have dirty faces and are all laughing. It suddenly flashes on A: why doesn't B realize C is laughing at her? —Heavens! *I* must be laughable. . . .

This is genuine mathematical reasoning But further, what has not got into the books so far as I know, there is an extension, in principle, to n ladies, all dirty and all laughing. There is an induction: . . .

But the induction step doesn't really work very well in this version of the puzzle; one needs to arrange for specific intervals during which each stage of the induction can be assumed to take place.

I should mention that Rick Hevener, the redoubtable error sniper who discovered many of the errors listed here, put forth a different argument to show that everyone (all the professors, or all the ladies with dirty faces) will realize the situation immediately. For example, in the case of three professors, Prof. 1, knowing of two errors, reasons that if he made no error, then Prof. 2 knows of only one error, and furthermore that Prof. 2 must conclude that Prof. 3 knows of no errors. But this can't be true once Prof. X has made his announcement, so Prof. 1 concludes that he must have made an error.

In general, for n professors, Prof. 1 believes that Prof. 2 believes that Prof. 3 believes that . . . Prof. n knows of no errors, which can't be true after Prof. X's announcement.

This might be the argument Littlewood had in mind, again presenting a problem. Does Prof. 1 conclude immediately that Prof. n has already concluded immediately that something's amiss? Should he wait a few seconds, or perhaps a few minutes? For those who would like to argue interminably about which answer is "correct", I would simply like to say that in reality all the professors will just say "yep, I knew that already" and forget about it!

Page 624, line 4:

$(-\infty, 1]$ should be $(-\infty, -1]$.

Page 624, Answer for 7(a), line 3:

$(-C/A)$ should be $(-C/B)$.

Page 632, at end of Problem 11(c) add:

"So the chandelier was at distance $16 \cdot (5/2)^2 = 100$ feet from the ceiling.

Page 646, Problem 1(xi), line 2:

$\frac{\cos x}{x}$ should be $\frac{\sin x}{x}$

Page 655, Problem 5(ix):

The second term in the equation for I should be $-\frac{2x+1}{3(x^2+x+1)}$

Page 658, Problem 7 (iii):

$c_i = (i+1)a_{i+1}$